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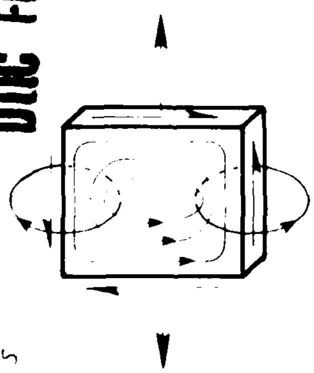

Departments of Theoretical & Applied Mechanics and Structural Engineering

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FINITE ELEMENT ANALYSIS OF COUPLED MAGNETOMECHANICAL PROBLEMS OF CONDUCTING PLATES

Kuan-Ya Yuan, Francis C. Moon, and John F. Abel

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In design problems for fusion reactor structures and other magnetically loaded devices, the field and the magnetic forces are usually calculated for static conditions only. The structural analysis is then handled as a separate problem. Recently, more attention is being given to the transient field problems, but little work has been done in understanding the nature of the dynamic magnetic forces and the—

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FINITE ELEMENT ANALYSIS OF COUPLED MAGNETOMECHANICAL PROBLEMS OF CONDUCTING PLATES[1]

Kuan-Ya Yuan,[2] Francis C. Moon,[3] and John F. Abel[4]

SUMMARY

In design problems for fusion reactor structures and other magnetically loaded devices, the field and the magnetic forces are usually calculated for static conditions only. The structural analysis is then handled as a separate problem. Recently, more attention is being given to the transient field problems, but little work has been done in understanding the nature of the dynamic magnetic forces and the mutual interactions between the field and the structure. This paper presents a study of the magnetic forces, the mutual interaction between fields, and the induced vibrations of long conducting plates. The finite element method is used for analysis. Some numerical results are presented.

INTRODUCTION

Magnetic forces result from the interactions between the external field and the induced current in a conducting body, assumed here to be nonferromagnetic. The phenomenon of currents induced by time-varying magnetic fields is governed by Faraday's induction law, and is called the

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"eddy current problem" in electromagnetism. The problem is a three-dimensional one for conducting plates with finite thickness. When the thickness of the plate is small compared to the penetration depth of the magnetic field, the induced current density across the thickness of the plate is approximately uniform. A stream function representation of the current may then be used to reduce the problem to a two-dimensional one. It is usually assumed that the externally applied magnetic field is not affected by the induced current. The total magnetic field may be decomposed into an externally applied part and a self-induced part due to the generation of the eddy current. The self-induced part may be determined by the Biot-Savart law, which involves integration of field quantities over the entire volume of the plate. An integro-differential equation may then result for the determination of the stream function. The integral terms represent the flux linkage of the eddy current density over different part of the plate and are called the "non-local" terms below. For infinitely long plates, a one-dimensional integro-differential equation may be derived for the calculation of the induced eddy current. Results of a study for steady-state, harmonic currents in long, rigid plates have been presented in Reference 1.

EQUATIONS AND FINITE ELEMENT FORMULATION

The basic equations for the coupled theory for linear nonferromagnetic plates are the Maxwell's equations

$$\begin{aligned} &\nabla \times \underset{\sim}{H} = \underset{\sim}{J} && \nabla \cdot \underset{\sim}{B} = 0 \\ &\nabla \times \underset{\sim}{E} = -\frac{\partial}{\partial t}\underset{\sim}{B} && \nabla \cdot \underset{\sim}{E} = 0 \end{aligned} \tag{1}$$

supplemented by Ohm's law

$$\underset{\sim}{J} = \sigma\,[\underset{\sim}{E} + \frac{\partial}{\partial t}\underset{\sim}{u} \times \underset{\sim}{B}] \tag{2}$$

and the linear equation of motion for the conducting plate

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = F + \underset{\sim}{n} \cdot \nabla \times \underset{\sim}{C} \tag{3}$$

where

$$F = \int_{-h/2}^{h/2} \underset{\sim}{n} \cdot (\underset{\sim}{J} \times \underset{\sim}{B})\, dz$$

$$\underset{\sim}{C} = \int_{-h/2}^{h/2} \underset{\sim}{n} \times (\underset{\sim}{J} \times \underset{\sim}{B})\, z\, dz \tag{4}$$

in which $\underset{\sim}{E}$, $\underset{\sim}{H}$, $\underset{\sim}{B}$, and $\underset{\sim}{J}$ are the electric field intensity, the magnetic field intensity, the induction, and the current density, respectively. $\underset{\sim}{u}$ is the displacement field and w the transverse deflection of the mid-surface of the plate. $\underset{\sim}{n}$ is the unit normal vector to the mid-surface. σ is the electrical conductivity, ρ the mass density, h the thickness of the plate, and D the bending rigidity of the plate. (A complete list of symbols used is given in the Appendix).

Using the stream function representation for the current, $\underset{\sim}{I} = \nabla \times (\psi \underset{\sim}{n})$ in which $\underset{\sim}{I} = h\underset{\sim}{J}$, and following a procedure suggested by Moon [2] for the calculation of the self-field, one obtains the following equations for long conducting plates (Fig. 1):

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial}{\partial t}\left[\sigma\mu_o \psi - \frac{\sigma\mu_o h}{2\pi} \int_o^{\ell} \frac{\psi(\xi)\, d\xi}{(\xi - x)^2 + \frac{1}{4}h^2}\right]$$

$$= \sigma h \left[\frac{\partial B_z^0}{\partial t} - \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial t} B_x^0\right)\right] \tag{5}$$

and

$$\rho \frac{\partial^2 w}{\partial t^2} + D \frac{\partial^4 w}{\partial x^4} = - \frac{\partial \psi}{\partial x} B_x^0 \tag{6}$$

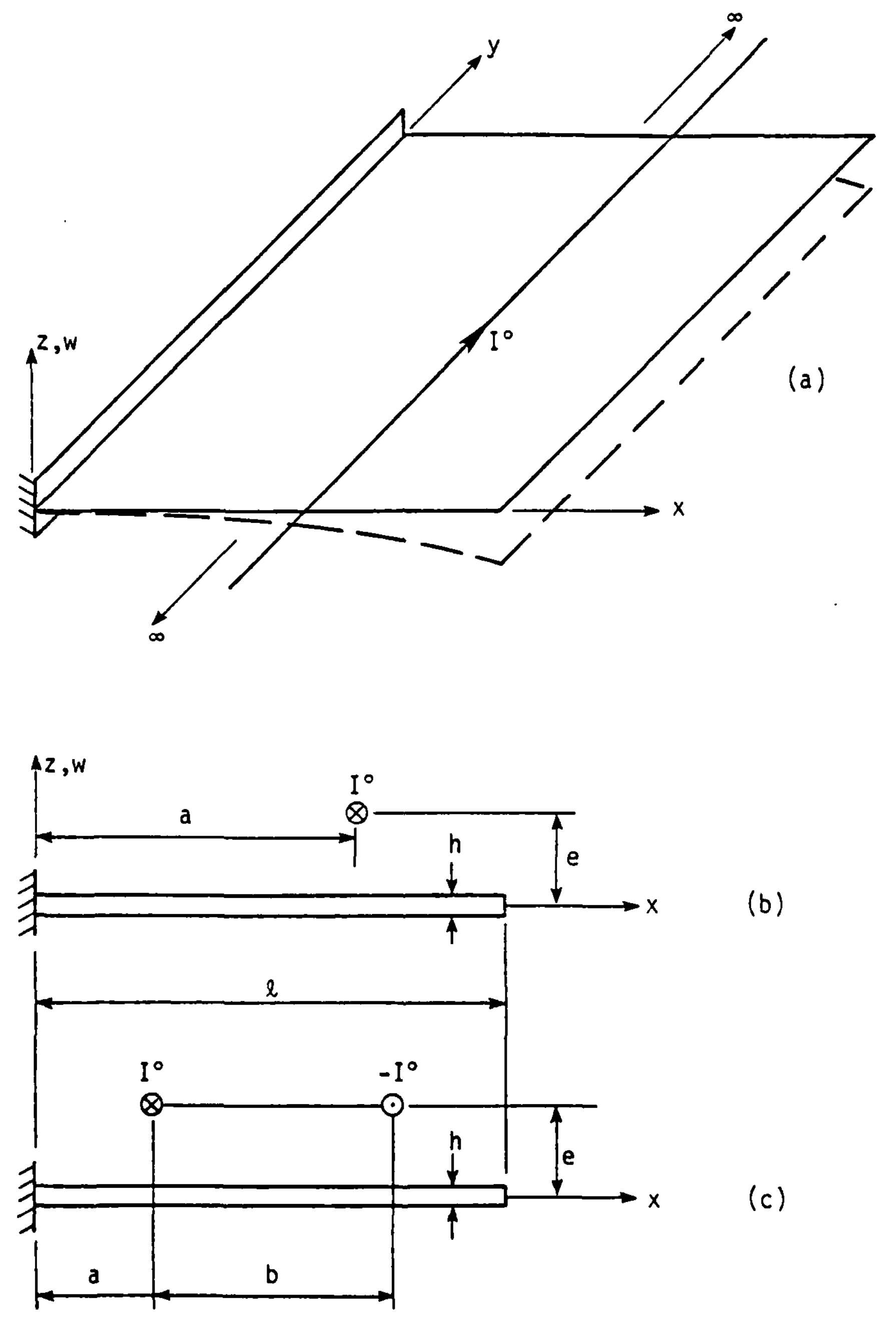

Figure 1. Long, cantilever conducting plate: (a) Isometric view, single wire, (b) Section, single-wire exciting coil, (c) Section, double-wire exciting coil.

in which the superscript 0 represents the applied (external) portion of the total field. The time-dependent terms on the left hand side of Eq. (5) represent the nonlocal effect of the self-field. In both equations the coupling effects appear on the right-hand side only.

The finite element Galerkin method is used to solve the coupled equations (5) and (6). ψ is approximated globally and locally by piece-wise linear models

$$\psi = \sum_{k=1}^{G} M_k \psi_k \text{ and } \psi = \sum_{k=1}^{2} N_k^E \psi_k \tag{7}$$

in which G is the total number of nodal points, M_k are the global interpolation functions generated from the local linear element shape functions N_k^E with the superscript E denoting the E<u>th</u> element. w is approximated by the usual cubic model.

$$w = \sum_{k=1}^{6} C_k^E w_k \tag{8}$$

The following set of linear algebraic equations for each element then results from Eqs. (5) and (6):

$$-\sum_{k=1}^{2} S_{jk} \psi_k - \sum_{k=1}^{2} P_{jk} \dot{\psi}_k + \sum_{k=1}^{G} Q_{jk} \dot{\psi}_k = R_j \tag{9}$$

in which

$$S_{jk} = \int_E \frac{dN_j^E}{dx} \frac{dN_k^E}{dx} dx$$

$$P_{jk} = \sigma\mu_o \int_E N_j^E N_k^E dx$$

$$Q^E_{jk} = \frac{\sigma\mu_o}{2\pi} \int_o^\ell M_k(\xi) \overset{E}{W}_j(\xi)\, d\xi$$

$$R_j = \sigma h \int_E N^E_j \left[\dot{B}^0_z - \frac{\partial}{\partial x} (\dot{w}B^0_x)\right] dx \tag{10}$$

where the weighting function is

$$\overset{E}{W}_j(\xi) = \int_E \frac{hN^E_j(x)}{(\xi-x)^2 + \frac{1}{4}h^2}\, dx. \tag{11}$$

Both $\overset{E}{W}_j(\xi)$ and Q^E_{jk} are integrated analytically. The integrations for Q^E_{jk} are carried out over the entire plate and result in a full, usually unsymmetric, matrix [3].

In matrix notation, the equations of motion are

$$[M]\{\ddot{w}\} + [K]\{w\} = \{F\} \tag{12}$$

in which

$$\{F\} = - \int_E \{C\} \frac{\partial\psi}{\partial x} B^0_x\, dx \tag{13}$$

The global matrix form of Eq. (9) is

$$[A]\{\dot{\psi}\} - [S]\{\psi\} = \{R\} \tag{14}$$

in which

$$[A] = - [P] + [Q] \tag{15}$$

In the general case, [A] is full and nonsymmetric.

The transient equations (14) and (12) are integrated using the following scheme.

$$\{\frac{1}{\Delta t}[A] - (1-\theta)[S]\}\{\psi\}_{t+\Delta t}$$

$$= \{\frac{1}{\Delta t}[A] + \theta[S]\}\{\psi\}_t + \theta\{R\}_t + (1-\theta)\{R\}_{t+\Delta t} \qquad (16)$$

and

$$\{[M] + \frac{(\Delta t)^2}{4}[K]\}\{\ddot{w}\}_{t+\Delta t}$$

$$= \{F\}_{t+\Delta t} - [M]\{\{w\}_t + \Delta t\{\dot{w}\}_t + \frac{(\Delta t)^2}{4}\{\ddot{w}\}_t\} \qquad (17)$$

Equations (16) and (17) form a weakly coupled set of equations. The forcing function {R} in (16) contains terms proportional to $\frac{\partial}{\partial x}(\dot{w}B_x^0)$. Its magnitude is small compared to the total induced eddy currents. The computation is performed by two more or less independent routines. The solution to the coupled system is advanced by sequentially executing these two routines. The coupling term in $\{R\}_{t+\Delta t}$ in Eq. (16) is estimated by temporal extrapolation.

EXAMPLES

Numerical simulation has been conducted for the magnetically induced vibrations of thin aluminum plates (conductivity $\sigma = 3.8 \times 10^7$ 1/ohm-m and wave speed $c_v = \sqrt{E/\rho} = 5090$ m/sec). Because of the widely different time response characteristics of the electromagnetic and mechanical fields, two levels of investigations have been carried out. To study the effects of the magnetic coupling of the eddy current density on the distribution and time variation of current and force, the plate is first assumed to be rigid. For the arrangement shown in Fig. 1(c), with ℓ = 485 mm, a = 94.31 mm, b = 314 mm, e = 9.922 mm and h = 2.381 mm, the transient analysis for a half-sine driving pulse in a double-wire

exciting coil gives the results shown in Fig. 2. The effect of the nonlocal terms is apparent and includes both a phase shift and a nonzero average pushing force, as predicted in the study for steady-state, harmonic current cases [1].

The coupled magnetomechanical effect is considered for the arrangement shown in Fig. 1(b), with ℓ = 427 mm, a = 334.6 mm, h = 2.11 mm, and e = 4h = 8.44 mm. The exciting wire is purposely placed above the nodal point of the second vibration mode of the cantilever plate. Because of the much sharper variation of the electromagnetic field variables, more field elements than beam elements are used in the analysis. The meshes of the two subsystems are made conformable by dividing each beam element into several equal-length field elements. The coupling terms in the two subsystems are evaluated by interpolation and numerical integration.

To have meaningful coupling between the electromagnetic and mechanical subsystems, a 3ms pulse duration in the exciting wire is chosen, half the period of the third vibration mode of the beam-plate (T_3 = 6ms, Δt = 0.075ms). The results of the analysis are shown in Figs. 3 and 4 for a driving current of $I°$ = 500 Amp. The $\dot{\underset{\sim}{u}} \times \underset{\sim}{B}$ term ("two-way coupling") demonstrates itself as a damping effect and has its maximum influence in the region close to the exciting wire. The time histories of eddy current, force, and transverse deflection at a point close to the exciting coil are shown in Figure 4. The force plot shows that a pulling force can be generated, and the displacement plot indicates that the energy transfer into the mechanical subsystem is reduced because of the $\dot{\underset{\sim}{u}} \times \underset{\sim}{B}$ effect. The nonlocal effect is significant and generates a net pushing force. The third mode component also shows

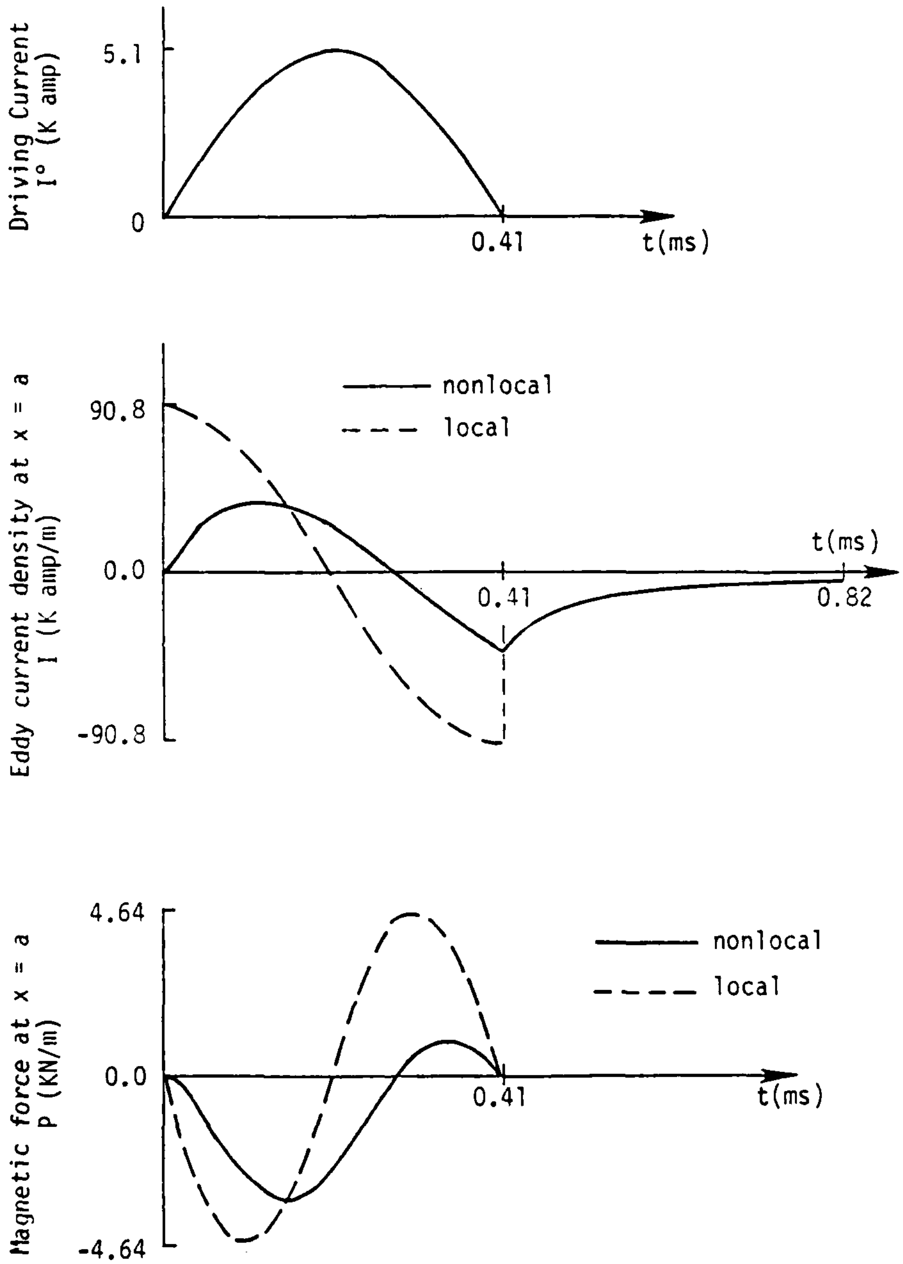

Figure 2. Comparison of local and nonlocal predictions of transient eddy current and magnetic force for a long plate subject to a half-sine pulse in a double-wire exciting coil.

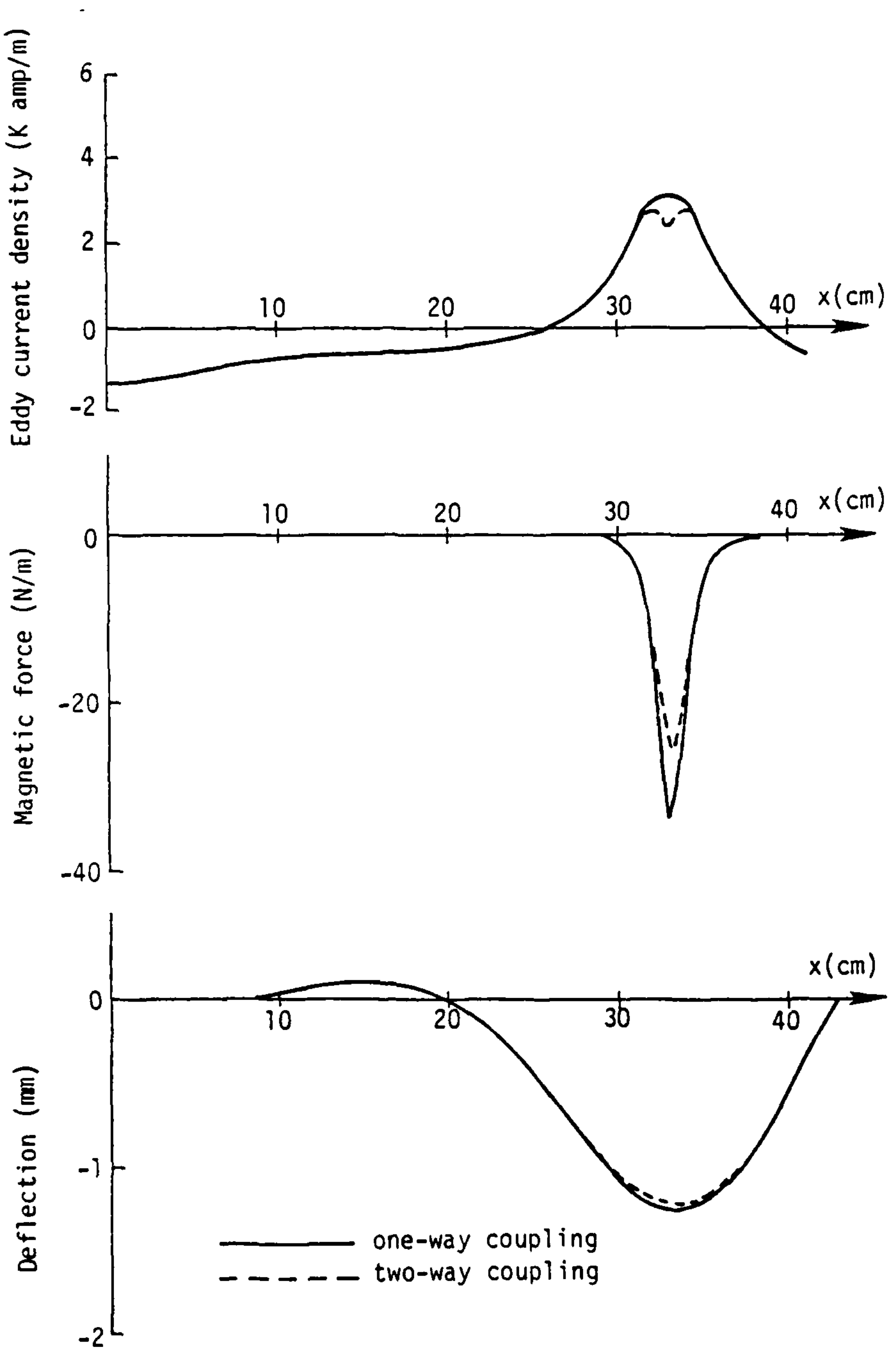

Figure 3. Nonlocal predictions of eddy current density, magnetic force, and transverse displacement at t = 0.9 ms for cantilever plate with single-wire exciting coil.

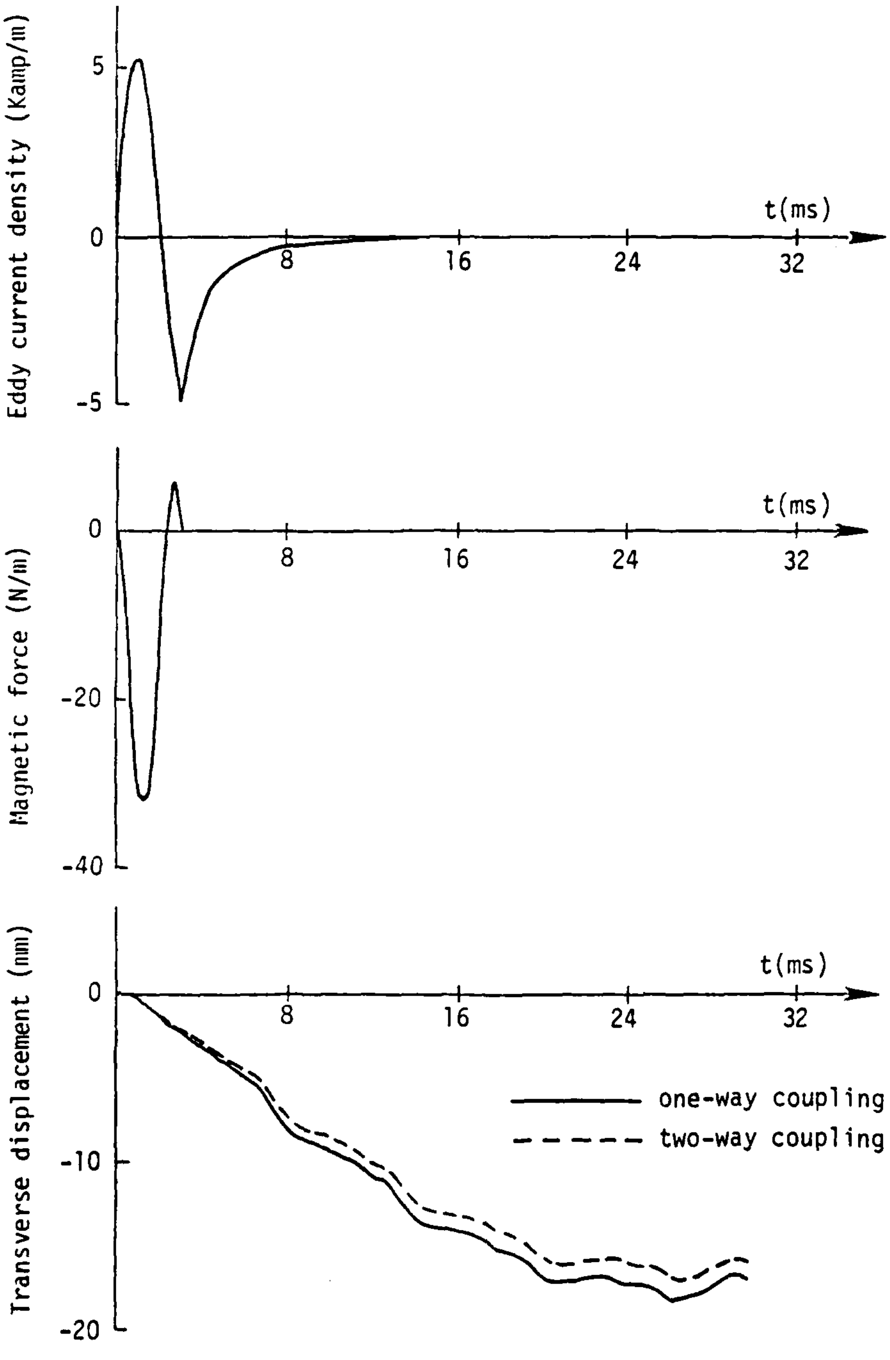

Figure 4. Nonlocal predictions of eddy current density, magnetic force, and transverse displacement at x ≅ 33 cm for cantilever plate with single-wire exciting coil.

up clearly in the displacement plot, superimposed on the first mode response (T_1 = 105ms).

CONCLUSION

A coupled electro-magneto-mechanical problem has been studied for the magnetically induced vibration of thin nonferromagnetic conducting plates. Some transient analysis results have been presented for the pulsed current problem. The nonlocal and $\dot{\tilde{u}} \times \tilde{B}$ couplings have been investigated. The results show that the nonlocal effect is essential for the calculation of force, and that the $\dot{\tilde{u}} \times \tilde{B}$ term acts as a damping effect in the coupled problem. Experience so far has been limited to simple pulsed currents. Work in progress includes experimental verification, further numerical studies, and extension to geometrically nonlinear effects of deformation. Moreover, cyclically pulsed exciting currents need to be examined as a possible source of structural instability.

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APPENDIX: LIST OF SYMBOLS

[A] electromagnetic field matrix

$\underset{\sim}{B}$ induction

D bending rigidity

$\underset{\sim}{E}$ electric field intensity

F magnetic force

h thickness of plate

$\underset{\sim}{H}$ magnetic field intensity

I° driving current in wire

$\underset{\sim}{I}$ current per unit length

$\underset{\sim}{J}$ current density

[K] stiffness matrix

ℓ width of plate

M_k global shape functions

[M] mass matrix

$\underset{\sim}{n}$ unit normal vector to the mid-surface

N_k^E, C_k^E linear and cubic element shape functions, respectively

[P],[Q],[S] electromagnetic field matrices

$\underset{\sim}{u}$ displacement

w transverse deflection of plate

W_j^E weighting functions

x,z,ξ orthogonal cartesian coordinates

μ_o permeability of free space

ρ mass density

σ electric conductivity

ψ stream function for eddy current

0 superscript 0 denotes externally applied quantities

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